**Lecturas Matemáticas**
*Volumen* **27** *(2006), páginas 19–20*

# A counterexample to a proposition of R. Mathews

Pablo A. Acosta Solarte
Universidad Distrital "Francisco José de Caldas", Bogotá

Abstract. Counterexamples to a proposition of R. Mathews are given.

*Key words and phrases.* Polynomials over finite fields, permutation polynomials, finite fields.

*2000 AMS Mathematics Subject Classification.*

Resumen. Se dan contraejemplos a una proposición de R. Mathews.

*Palabras y frases clave.* Polinomios sobre cuerpos finitos, polinomios de permutación, cuerpos finitos.

In [2] it is quoted that R. Mathews in his Ph. D. Dissertation [1] proves the following proposition:

**Proposition.** *Let*

$$f_k(X) = \sum_{j=o}^{[k/2]} \binom{k-j}{j} (-1)^j X^{k-2j} \in \mathbb{F}_q[X] \ ,$$

*where $q$ is a power of a odd prime $p$. Then $f_k(X)$ is a permutation polynomial if $k+1 \equiv \pm 2 (\text{mod } m)$, for $m = p$, $(q-1)/2$ and $(q+1)/2$.*

We contend that this proposition as stated is false. Indeed, let $k = 11$, $p = q = 5$. If $m = 5$, then $11 + 1 \equiv 2(\text{mod } 5)$. So the conditions in the Proposition are satisfied. However,

$$f_{11}(X) = X^{11} - 10X^9 + 36X^7 - 56X^5 + 35X^3 - 6X \in \mathbb{F}_5[X]$$

is not a permutation polynomial since $f_{11}(0) = 0$ and $f_{11}(1) = 0$. Also if $q = 9$ and we put $m = (q-1)/2 = 4$ and $k = 17$, the polynomial

$$f_{17}(X) = X^{17} - 16X^{15} + 105X^{13} - 364X^{11} + 715X^9 - 792X^7 + 462X^5 - 120X^3 + 9X$$

satisfies $f_{17}(0) = f_{17}(1) = 0$. We remark that in this case $f_{17}(0) = f_{17}(1) = 0$ no matter the value of $q$.

## References

[1] R. Mathews. *Permutation polynomials in one and several variables.* Ph. D. Thesis, University of Tasmania, 1982.

[2] M. Henderson & R. Mathews. *Permutation properties of Chebyshev polynomials of the second kind over a finite field*. Finite Fields and Appl. **1** (1995), 115–125.

(Recibido en enero de 2005. Aceptado para publicación en agosto de 2005)

Universidad Distrital "Francisco José de Caldas"
Bogotá, Colombia.
*e-mail:* `paacostas@udistrital.edu.co`
`pacostasolarte@yahoo.es`