

## AMDS 2014 Scientific program

### Schedule

	Mon. 10/6	Tue. 10/7	Wed. 10/8	Thu. 10/9	Fri. 10/10
8:00-9:00	J.-P. Ramis	K. Yagasaki		Y. Brezhnev	A. Maciejewski
9:00-10:00	S. Rueda	O. Lisovyi		T. Combet	A. Mironov
10:00-10:30	Coffee Break	Coffee Break		Coffee Break	Coffee Break
10:30-11:30	M. Barkatou	J.-A. Weil		M. Przybylska	P. Vanhaecke
11:30-12:30	G. Jiménez  T. Dreyfus	O. Christov  F. Heiderich		T. Stachowiak  L. Benítez	H. Baran  A. Yakhno
12:30-15:00	Lunch  Break	Lunch  Break		Lunch  Break	Lunch  Break
15:00-16:00	J.-P. Ramis & J. Sauloy	R. Perez-Marco		M. van der Put	J. Delgado  P. Vojčák
16:00-17:00	E. Paul	N. Ibragimov		P. Olver	A. Castañeda  M. Wibmer
17:00-18:00	C. Pantazi  A. Reyes	Opening ceremony		E. Avdonina  M. Machado	A. Eloy  G. Vergara

## Plenary Talks (50 Minutes)

1. Jean-Pierre Ramis, Université Paul Sabatier, France

### **Dynamics on wild character varieties and Painlevé equations.**

Abstract: We will describe a work in progress (in collaboration with E. Paul and J. Rebelo). Our main purpose is to imitate the work of the Japanese school (Iwasaki...) and of Cantat-Loray about the Painlevé equation PV I in order to prove that the Malgrange groupoid for all the “generic” Painlevé equations is “the biggest possible”. Today it is more a program than complete results... The idea is to define a “wild dynamic” on each Painlevé equation and to translate it into an algebraic dynamic on the corresponding “wild character variety”. Then it will become possible to study explicitly this algebraic dynamic with the hope that it is “very rich”. Finally we will try to derive similar results for the Painlevé wild dynamic. We will describe a general picture for iso-irregular deformations of linear differential equations admitting as gauge group a reductive group  $G$ . We get in this case an action of a “wild braid group” on the corresponding wild character variety (Boalch). We will explain in more details what is happening for Painlevé equations (in particular a way to compute the character varieties slightly different from the existing methods) and why wild braid groups are not sufficient for our purpose.

2. Sonia L. Rueda, Technical University of Madrid, Spain

### **Differential resultants and Burchinal-Chaudy polynomials**

Abstract: Differential elimination techniques play an important role in the algorithmic treatment of algebraic differential equations. A matrix representation of sparse differentials resultant is the basis for efficient computation algorithms, whose study promises a great contribution to the development and applicability of differential elimination techniques. In this talk, it will be shown how sparse linear differential resultant formulas provide bounds for the order of derivation, even in the nonlinear case, and they also provide (in many cases) the bridge with results in the nonlinear algebraic case. As an important application, some results on the use of differential resultants to understand the spectral curve of two ordinary differential operators will be presented.

3. Moulay Barkatou, Université de Limoges, France

### **On Subanalytic solutions of linear difference equations**

Abstract: We consider linear difference equations with polynomial coefficients over  $\mathbb{C}$  and their solutions in the form of sequences indexed by the integers (sequential solutions). We investigate the  $\mathbb{C}$ -linear space of subanalytic solutions, i.e., those sequential solutions that are the restrictions to  $\mathbb{Z}$  of some analytic solutions of the original equation. It is shown that this space coincides with the space of the restrictions to  $\mathbb{Z}$  of entire solutions and that the dimension of this space is equal to the order of the original equation. Subanalytic solutions have applications in computer algebra. We show that some implementations of certain well-known summation algorithms in existing computer algebra systems work correctly when the input sequence is a subanalytic solution of an equation or a system, but can give incorrect results for some sequential solutions.

4. Jean-Pierre Ramis and Jacques Sauloy, Université Paul Sabatier, France

### **TBA**

Abstract: TBA

5. Emmanuel Paul, Université Paul Sabatier, France

**Dynamics on character varieties**

Abstract: The set of the monodromy representations of a 2-rank regular linear connection on the punctured sphere defines a character variety on which the group of exterior automorphisms of the fundamental group acts. This dynamic is related to Painlevé VI equation since this one encodes isomonodromic deformations. We will present the extensions of this description to the irregular cases (work in progress). This is a joint work with JP Ramis, J. Rebelo.

6. Kazuyuki Yagasaki, Kyoto University, Japan

**Analytic and algebraic conditions for bifurcations of homoclinic orbits in reversible systems**

Abstract: Bifurcations of homoclinic orbits to hyperbolic saddles or saddle-centers in reversible systems are studied analytically by Melnikov-type methods and algebraically by differential Galois theory. An example is given to illustrate the theoretical results.

7. Oleg Lisovyi, Université de Tours, France

**Painlevé transcendents and conformal blocks**

Abstract: I will discuss a recently discovered relation of Painlevé equations and representation theory of the Virasoro algebra. It will be explained how the Riemann-Hilbert problem associated to isomonodromic deformations of rank  $2$  linear systems with  $n$  regular singular points on  $\mathbb{P}^1$  can be solved by taking suitable linear combinations of Liouville conformal blocks at  $c=1$ . This implies a similar representation of the isomonodromic tau function. In the case  $n=4$ , it provides the general solution of the Painlevé VI equation in the form of combinatorial sum over pairs of Young diagrams. Analogous solutions of Painlevé V and III can be obtained in terms of irregular conformal blocks.

8. Jacques-Arthur Weil, Université de Limoges, France

**TBA**

Abstract: TBA

9. Ricardo Pérez-Marco, CNRS and Université de Paris XIII

**TBA**

Abstract: TBA

10. Nail Ibragimov, Ufa State Aviation Technical University, Russia

**Integration of dynamical systems using nonlinear superposition**

Abstract: In the present lecture we discuss an approach to integration of nonlinear non-autonomous dynamical systems. The method is applicable to the systems admitting a nonlinear superposition. The integration technique is based on the Lie algebra structures specific for nonlinear superpositions.

11. Yurii Brezhnev, Tomsk University, Russia

**On a quantization of Jacobian theta-functions**

Abstract: The famous theta-functions, including the 1-dimensional Jacobi's theta-functions, appears in numerous areas of mathematical physics as building blocks for representation of solutions. They also have great interest in their own rights because of their rich structural properties. Recently we found some new such properties. In particular, Jacobi's functions can be defined by a system of ordinary differential equations, i.e. by a dynamical system. Such a property allows us to state the problem of their quantization in a classical way. It was found that the problem has a particular solution which is a nontrivial extension of the classical quantization of a rigid body (Euler top). We exhibit the complete set of integrals of motion for

this new system and discuss their role in a quantization procedure. Surprisingly, the accurate statement of the problem requires an extension of the set of observables and the next theta-functional generalization to the 3-dimensional Euler's dynamical system admits a complete quantization procedure including the representation for observables by differential operators; the bracket is noncanonical. Corresponding Hamiltonian has a continuous spectrum with an infinite band structure. It is described by the Mathew equation: the periodic sine-type potential in the Schrodinger equation.

12. Thierry Combot, Université de Dijon, France

### **Towards a complete classification of integrable homogeneous polynomial potentials in the plane**

Abstract: We first present the numerous results of non-integrability (with rational or meromorphic first integrals) of such potentials and a list of known integrable potentials. As this list is probably complete, further work is necessary to prove that all other potentials are non-integrable. Thanks to a relation of Maciejewski and Przybylska, for any fixed degree  $k$ , it is algorithmically possible to make a classification of integrable homogeneous potentials in the plane except for a few exceptional cases. We can now expect to obtain soon a complete classification for all  $k$ . We will thus present some new ideas to solve the two remaining problems: to prove non-integrability of these remaining exceptional cases, and to generalize the approach without needing to fix the degree  $k$ . The exceptional cases are related to potentials without Darboux points (solutions of  $V'(c) = \alpha c$ ), for which the classical approach using the Morales-Ramis Theorem is completely ineffective. However, a perturbative version of this Theorem seem to be able to remove most of these cases. This would produce a classification for small  $k$ . On the other hand, for sufficiently large enough  $k$ , it seems possible to bound the eigenvalues at Darboux points, which are crucial quantities in this integrability analysis. Higher variational equations with the Morales-Ramis-Simo Theorem can then be effectively used at any order, which would probably remove most cases. To conclude, we will discuss how much these approaches can be applied for rational potentials, and which results could be expected.

13. Maria Przybylska, University of Zielona Góra, Poland

### **Differential Galois obstructions to the integrability of certain constrained n-body problems**

Abstract: Class of various systems of mass points interacting gravitationally whose motion is subjected to certain holonomic constraints is considered. The complicated behaviour of their trajectories is illustrated by means of Poincare cross sections. For some models one can prove their non-integrability using properties of differential Galois group of variational equations along certain particular solutions. Amazing relation of some of these systems with anisotropic Kepler problem is shown. These results were obtained in collaboration with Wojciech Szuminski.

14. Marius van der Put, University of Groningen, Netherlands

### **The Stokes phenomenon and some applications**

Abstract: In the first part of the lecture we aim to give a precise definition of the Stokes maps for linear differential equations over the complex numbers. Then we apply this to produce moduli spaces of differential equations in relation with Painlevé equations. Further we explain Stokes matrices for quantum differential equations and their computation by purely algebraic means. The results of this lecture were obtained in cooperation with Jaap Top and John Alexander Cruz-Morales.

15. Peter Olver

### **Moving frames, variational problems, and geometric flows**

Abstract:

I will begin with an overview of the new, equivariant approach to the method of moving frames, and some of its applications, including the classification of differential invariants and the construction of signatures to

solve equivalence problems arising in computer vision: object recognition and symmetry detection. I will then discuss how the moving frame invariant variational bicomplex construction solves a problem arising in the calculus of variations: how to directly construct the differential invariant form of the Euler--Lagrange equations of an invariant variational problem. Finally, the invariant variational machinery will be used to analyze the evolution of differential invariants under invariant geometric flows, with applications to the evolution of differential invariant signatures. Many of these flows lead to integrable soliton equations, but the precise mechanism is not well understood. I will conclude by presenting recent attempts to analyze the Poisson structures and integrability of such flows.

16. Andrzej J. Maciejewski, University of Zielona Góra, Poland

**Non-standard problems in applications of differential Galois theory to integrability studies**

I present several applications of differential Galois theory in some rather not typical cases. Hamiltonian as well as non-Hamiltonian systems will be considered. A few interesting results obtained with my collaborators will be shown.

17. Andrey Mironov, Sobolev Institute of Mathematics, Russia

**Commuting rank two differential and difference operators**

Abstract: We discuss the problem of finding commuting higher rank differential and difference operators. In the case of hyperelliptic spectral curves equations which are equivalent to the Krichever-Novikov equations on Tyurin parameters are obtained. With the help of these equations examples of differential and difference operators of rank two corresponding to spectral curves of higher genus are constructed.

18. Pol Vanhaecke, Université de Poitiers, France

**TBA**

Abstract: TBA

## Contributed Talks (20 minutes)

1. Germán Jiménez, Universidad del Norte, Colombia

**Morales-Ramis Theory: statements and examples**

Abstract. Morales-Ramis theory is considered the Galois theory in the context of dynamical systems. In case of Hamiltonian systems, it relates two kind of different notions of integrability: the integrability of Hamiltonian through Liouville-Arnold theorem and integrability of the variational equations through differential Galois theory (Galois theory for linear differential equations). In this talk, a short survey concerning to some results in Morales-Ramis theory it will be presented. Some examples are presented, including second order variational equation and integrable systems in where the reciprocal of Morales-Ramis theory is not true. This is joint work with P. Acosta-Humánez.

2. Thomas Dreyfus, Université de Toulouse, France

**Difference equations on elliptic curves**

Abstract: Let us consider a difference equations of order two with coefficients that are meromorphic over an elliptic curve. To such equation, we may associate a group, the Galois group, that measures the algebraic relations between the solutions. It is an algebraic subgroup of invertible complex matrices. The goal of this talk is to give necessary and sufficient conditions for the reducibility and the imprimitivity of the Galois group. This is joint work with Julien Roques.

3. Chara Pantazi, Universitat Politecnica de Catalunya, Spain

#### **Polynomial vector fields and Picard-Vessiot theory**

Abstract: We study the integrability of polynomial vector fields using Galois theory of linear differential equations when the associated foliations is reduced to a Riccati type foliation. In particular we obtain integrability results for some families of quadratic vector fields, Lienard equations and equations related with special functions such as Hypergeometric and Heun ones. The Poincaré problem for some families is also approached. This is a joint work with P. Acosta-Humánez, J.T. Lázaro and J. Morales-Ruiz.

4. Armando Reyes, Universidad de los Andes and Universidad de la Salle, Colombia

#### **Involutive Partial Differential Equations and Skew PBW Extensions**

Abstract: We present the notion of involutive partial differential equation. We will show that involution is a stronger concept than formal integrability, and we will study its purely algebraic notion considering a natural polynomial structure lies hidden in the inner geometry of the jet bundle. It allows us to associate with any system of differential equation a module, its symbol module, over a polynomial ring in as many variables as there are independent variables. Hence we illustrate some results of these equations using a Gröbner basis theory of a kind of noncommutative algebras known as skew PBW extensions which includes algebras of interest for modern mathematical physicists such that group rings of polycyclic-by-finite groups, Ore algebras, operator algebras, diffusion algebras, quantum algebras and others.

5. Ognyan Christov, Sofia University, Bulgaria

#### **Non-integrability of a fourth-order Painlevé equation in Liouville sense**

Abstract: We study the equation

$$w^{(4)} = 5w''(w'^2 - w') + 5w(w')^2 - w^5 + (\lambda z + \alpha)w + \gamma,$$

which is one of the higher-order Painlevé equations (i.e. equations in the polynomial class having the Painlevé property). This equation appears as a group-invariant reduction of the modified Kaup-Kupershmidt (or Sawada-Kotera) equation, A. Hone, N. Kudryashov. Then it appears as equation F-XVIII in the classification, made by C. Cosgrove of all fourth- and fifth-order equations with Painlevé property. It is also studied by V. Gromak from different points of view. Like the classical Painlevé equations, this equation admits a Hamiltonian formulation, Bäcklund transformations and families of rational and special functions. For instance, when  $\lambda = 0$ ,  $\gamma \neq 0$  it can be solved via hyperelliptic functions, which become elliptic ones when  $\lambda = \gamma = 0$ . When  $\gamma = -\lambda/2$ ,  $w(z)$  can be expressed in terms of two Painlevé I solutions. Further, we assume that  $\lambda \neq 0$ . We prove that this equation considered as a Hamiltonian system with parameters  $\gamma/\lambda = 3k$ ,  $\gamma/\lambda = 3k - 1$ ,  $k \in \mathbb{Z}$ , is not integrable in Liouville sense by means of rational first integrals. To do that we use Ziglin - Morales-Ruiz - Ramis approach.

6. Florian Heiderich, National Research University HSE, Russia

#### **Towards a non-commutative Picard-Vessiot theory**

Abstract: André unified the previously independent Picard-Vessiot theories for linear differential and difference equations in 2001. The use of generalized derivations in André's work called for a possible application to broader classes of linear functional equations. Recently, Saito and Umemura gave an example of a Picard-Vessiot extension for a linear functional equation involving a skew-derivation that gives rise to a quantum group as Galois group. The aim of this talk is to present a general framework for a Picard-Vessiot theory for functional equations involving a wide class of operators including skew-derivations.

7. Tomasz Stachowiak, Center for Theoretical Physics, Poland

### **A connection between the Galois group and quantization in the Bargmann representation**

Abstract: The differential Galois theory provides an extremely useful formulation and tools for characterizing and finding explicit or closed-form solutions of differential equations, but this notion does not always coincide with solutions that are valid from physical point of view. However, there are some models of quantum mechanics, which naturally lead to linear systems over the complex plane, where those two aspects seem to be linked. This is achieved through the Bargmann representation in the space of entire functions. In this presentation I will describe some properties of the Galois group of such systems, and the conditions of quantization that follow from physical considerations. In particular I hope to elucidate the role of the Stokes matrices in connecting the two facets of the problem.

8. Luis Benítez, Universidad de Antioquia, Colombia

#### **Infinitesimal generators of evolution families**

The nonlinear resolvent,  $p$ -monotony and initial value problems were used to study infinitesimal generator of one-parameter semigroup in  $N(D)$ , the set of all  $p$ -nonexpansive mappings on the unit disk. In this talk, the concepts of infinitesimal generator of evolution families in  $N(D)$  is established. Under an additional property and using both the nonlinear resolvent and  $p$ -monotony, a characterization for infinitesimal generator of evolution families in  $N(D)$  is obtained.

9. Elena D. Avdonina and Nail H. Ibragimov, Ufa State Aviation Technical University

#### **Conservation laws and new solutions of anisotropic diffusion equations**

Abstract: Nonlinear mathematical models of heat conduction in anisotropic media with an external source are considered. A class of nonlinearly self-adjoint diffusion equations is identified. Conservation laws associated with symmetries are constructed for self-adjoint equations. Particular exact solutions are computed by the method of conservation laws. It is shown that the obtained solutions are different from group invariant solutions.

10. Maximiliano Machado, Universidad Veracruzana and Universidad de Ibagué, México / Colombia

#### **An application Abel's equation of first kind in a model of anaerobic digestion for production of biogas**

Abstract: We consider a nonlinear mathematical model for the study of anaerobic digestion process. This process consists for hydrolysis, acidogenesis, acetogenesis y methanogenesis, of which employ only the last two. We decompose the original system of nonlinear ODEs into subsystems. For these subsystems, we prove existence of lower and upper solutions in reverse order with respect a part of the variables. The upper and lower solutions are constructed in analytical form. Connection between the upper solutions of subsystem for feeding bacteria with solutions of Abel's equation of the first kind is found. We study different analytical solutions of Abel's equation implementing proposal in \cite{Salinas}.

**Keywords:** Cauchy problem, lower-upper solution, reverse order, Abel equation, anaerobic digestion, biogas.

This is a joint work with P.B. Acosta-Humánez and A.V. Sinitsyn.

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11. Hynek Baran, Silesian University in Opava, Czech Republic





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%\label{l_star_Veronese_web_eq}
\end{eqnarray}
with  $\alpha \neq 0, 1$ , the universal hierarchy equation,
\cite{AlonsoShabat2004,Pavlov2003},
\begin{equation}
u_{yy} = u_y \, u_{tx} - u_x \, u_{ty}, \quad
\label{universal_hierarchy_eq}
v_{yy} = u_y \, v_{tx} - u_x \, v_{ty} + 2 \, (u_{ty} \, v_x - u_{tx} \, v_y),
\end{equation}
and for the 4-dimensional analogue of \ref{Ovsienko_eq}, \cite{Morozov2013},
\begin{equation}
u_{ty} = u_z \, u_{xy} - u_y \, u_{xz}, \quad
\label{rdDym4D_eq}
v_{ty} = u_z \, v_{xy} - u_y \, v_{xz} + 2 \, (u_{xz} \, v_y - u_{xy} \, v_z),
\end{equation}
introduced in \cite{AlonsoShabat2004}. We show that these systems have local
symmetries of the third order. Equations \ref{Ovsienko_eq}, \ref{Pavlov_eq} and \ref{Veronese_web_eq}
belong to the families of pde s
\begin{eqnarray}
u_{ty} \, \&\& u_x \, u_{xy} + \kappa u_y \, u_{xx},
\label{rdDym_eq} \\
u_{yy} \, \&\& u_{tx} + \left( \frac{1}{2} \, (\kappa + 1) \, u_x^2 + u_y \right) \, u_{xx} +
\kappa u_x \, u_{xy},
\label{rmdKP_eq} \\
u_{xy} \, \&\& \frac{\alpha \, u_x \, u_{ty} + \beta \, u_y \, u_{tx}}{u_t}
\label{generalized_Veronese_web_eq}
\end{eqnarray}
and are featured as the only representatives of their families whose zero-curvature representation have
a non-removable parameter. This is a joint work with O. I. Morozov and P. Voj{\v{c}}{\l{a}}k
\keywords{Nonlinear partial differential equation, integrability, symmetry, conservation law, recursion
operator, JETS, distributed computing}
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12. Alexander Yakhno, University of Guadalajara, Mexico

**Conservation laws of three-dimensional perfect plasticity equations under von Mises yield criterion**

Abstract: For the first time the conservation laws for von Mises plasticity equations in three dimensions, as well as for plane stress equations are given. In the plane case conservation laws are used to construct characteristics for Cauchy problem. For the system of the plane strain, the conservation laws are used to solve the free boundary problem for any convex smooth contour loaded with constant normal and zero tangential stresses. The talk is based on the published article *{it doi:10.1155/2013/702132}*

13. Joaquín Delgado, Universidad Autónoma Metropolitana, Iztapalapa, México

**Lagrangian form of complex Ginzburg-Landau equation**

Abstract: The cubic complex Ginzburg-Landau equation (CGLE),

$$A_t = \epsilon A + (b_1 + i b_2) A_{xx} - (b_3 - i c_3) |A|^2 A - (b_5 - i c_5) |A|^4 A$$

describes weakly nonlinear behavior of dissipative systems. Here  $A(t,x)$  is a complex amplitude,  $t \in \mathbb{R}$ ,  $x \in \mathbb{R}^d$ , and all the parameters are real. For  $\epsilon = b_1 = b_3 = 0$  it reduces to the non-linear Schrödinger equation (NLS). For the  $d=1$  dimensional case, the inverse scattering method has been one of the standard methods to look for integrability of nonlinear partial differential equations as the NLS. One method relies on the ansatz  $A(t,x) = e^{i(\omega t - v(x-ct))}$  which reduces CGLE to a system of  $n=2$  second order ODEs for the real and imaginary parts of  $v = a + i b$ . The question as if this system is Lagrangian is important, as if the answer is positive, a finite-dimensional Hamiltonian form of the equations is possible via Legendre transformation. This question is at the heart of what is known as the *{it inverse problem of the calculus of variations}*, that was solved and classified for the case  $n=2$  by Jesse Douglas, the more complicated case of  $n>2$  requires the solution of an overdetermined systems of linear partial differential equations. This has been tackled by what is known as "geometric calculus" or Janet-Requier theory. We found that CGLE admits a *{it complex Lagrangian form}* and we exhibit some other example where the question of a complex formulation of a type of Arnold-Liouville theorem would be

interesting. At no surprise for  $d=1$ , we present the particular case of the reduction of the NLS under the aforementioned ansatz, as a complete integrable Hamiltonian system. We show how different type of soliton solutions arise as a consequence of quasiperiodic motion. For particular perturbations of the NLS we get soliton solutions of the CGLE by a direct application of KAM theorem. We also present recent results of existence and scattering of the full NLS. This is a joint work with Cristi Darley.

14. Petr Vojčák, Silesian University in Opava, Czech Republic

### Nonlocal structures for the Krichever--Novikov equation

Abstract: Integrable systems in addition to local symmetries usually possess infinite hierarchies of (shadows of) nonlocal symmetries. It can be even argued that precisely such nonlocal hierarchies are the most common feature of known today integrable partial differential systems in any number of independent variables.

The Krichever--Novikov (KN) equation

$$u_t = u_{xxx} - \frac{3}{2} \frac{u_{xx}^2}{u_x} + \frac{P(u)}{u_x},$$

where  $P(u)$  is a fourth-order polynomial, has first appeared in Krichever in connection with the study of finite-gap solutions of the Kadomtsev--Petviashvili equation which has plenty of physical applications from plasma physics to fluid dynamics, see e.g. Novikov and references therein. The KN equation (KNEQ) is the simplest known integrable one-field elliptic model, see Dem Sok.

Surprisingly enough, it was not known relatively recently whether the KN equation (KNEQ), which is well known to be integrable and possesses infinitely many local generalized symmetries and two recursion operators, see e.g. Dem Sok and references therein, has any (shadows of) nonlocal symmetries at all. Note that for many equations it is possible to obtain the shadows of nonlocal symmetries by applying their recursion operators to the scaling symmetries. However, the KN equation (KNEQ) has no scaling symmetry, so this approach does not work. One could also try to construct nonlocal variables as potentials for conservation laws and subsequently look for (shadows of) nonlocal symmetries depending on these variables; however, this method also gave no results for the equation in question.

This talk presents a solution of this problem Voj. We construct new infinite hierarchies of shadows of nonlocal symmetries, shadows of nonlocal cosymmetries and nonlocal Hamiltonian structures for the KN equation (KNEQ) using the inverse  $\mathcal{R}_1^{-1}$  of the fourth-order recursion operator of the latter. Moreover, we also tackle the problem, which was pointed out in Dem Sok, of how to apply the composition  $\mathcal{R}_2 \circ \mathcal{R}_1^{-1}$ , where  $\mathcal{R}_2$  is the sixth-order recursion operator for the KN equation (KNEQ), to the known symmetries of the equation in question.

Keywords: Krichever--Novikov equation, recursion operators, nonlocal symmetries, nonlocal cosymmetries, nonlocal Hamiltonian structures

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15. Álvaro Castañeda, Universidad de Chile, Chile

### **Almost Global Stability of Autonomous and Nonautonomous Dynamical Systems**

Abstract: We study the global asymptotic stability of the origin for the continuous and discrete dynamical system associated to polynomial maps in  $\mathbb{R}^n$  (especially when  $n = 3$ ) of the form  $F = \lambda I + H$ , with  $F(0) = 0$ , where  $\lambda$  is a real number,  $I$  the identity map, and  $H$  a map with nilpotent Jacobian matrix  $JH$ . We distinguish the cases when the rows of  $JH$  are linearly dependent over  $\mathbb{R}$  and when they are linearly independent over  $\mathbb{R}$ . In the linearly dependent case we find non-linearly triangularizable vector fields  $F$  for which the origin is globally asymptotically stable singularity (resp. fixed point) for continuous (resp. discrete) systems generated by  $F$ . In the independent continuous case, we present a family of maps that have orbits escaping to infinity. Finally, in the independent discrete case, we show a large family of vector fields that have a periodic point of period 3.

16. Michael Wibmer, RWTH Aachen, Germany

### **Difference algebraic groups**

Abstract: Difference algebraic groups, i.e., groups defined by algebraic difference equations occur as Galois groups of linear differential and difference equations depending on discrete parameters. I will explain some structural properties of these groups and show how these results can be used in the study of special functions.

17. Anton Eloy, Université Paul Sabatier, Toulouse

### **Galois group of q-difference equations**

Abstract: In this talk we'll briefly explain the computation by Ramis and Sauloy of the universal Galois group of q-difference equations with integral slopes and the generalization by Bugeaud to equations with arbitrary slopes. We'll also approach the density theorems of q-Galois group and the limits at present of the knowledge about these groups.

18. Gabriel Vergara, Universidad del Atlántico

### **Relations between group actions and quasi-isometries**

The main objective of this talk is to study the concept of a group action on metric spaces and topological spaces and from this fact, highlight some relationships that exist between the group action on these spaces and the concept of quasi-isometries. This is motivated by the reason that at a basic level in a course on group theory only the action of a group on a set is studied, but no issues as what types of geometrical properties of space are preserved under quasi-isometry addressed. We also describe how to build a presentation for an arbitrary group  $G$ , which acts by homeomorphisms on a simply connected topological space  $X$ . Finally we will make some comments regarding the following results:

i) If  $X$  is a simply connected length space and  $G$  acts cocompactly by isometries on  $X$ , then  $G$  has a finite presentation.

ii) If  $G_1$  and  $G_2$  are groups,  $A_1$  and  $A_2$  are finite sets and if  $G_1$  is quasi-isometric to  $G_2$  and  $G_2$  has a finite presentation  $\langle A_2 \mid R_2 \rangle$ , then  $G_1$  has a finite presentation  $\langle A_1 \mid R_1 \rangle$ .

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## Posters

1. Álvaro Castañeda, Universidad de Chile, Chile

### **Some Results on Markus-Yamabe Conjectures**

It is well known that if a linear nonautonomous system is globally asymptotically stable, there exists a corresponding density function. In this work, we extend this result to a family of quasilinear systems, provided the existence of a  $C^2$  preserving orientation diffeomorphism between the solutions of both systems. The construction of such diffeomorphism extends a linearization result in a particular case and allows to generalize a method developed to construct a density function in the autonomous case.

2. Edward Páez, Universidad Sergio Arboleda, Colombia

### **Comparative numerical study based on perturbed trajectories solutions**

Abstract: At the present time is important for us the study of Low Earth Orbit Satellites (LEO), due to the project, Libertad II of Sergio Arboleda University, an artificial satellite.\

The trajectory of a LEO satellite can be derived with a model based on Kepler's equation \cite{Vallado}:  $(\ddot{r} = -\frac{\mu}{r^3} + a_{\text{perturb}})$  where we add a term due to the acceleration made by atmospheric drag, solar radiation, and Earth gravity harmonics.\

We find numerical solutions of a LEO satellite under the effect of these perturbations with two methods: the first one, by the direct calculation of the variation of Classic Orbital Elements (COE) (a (semimajor axis), e (eccentricity), i (inclination),  $\omega$  (argument of perigee),  $\Omega$  (longitude of ascending node) and  $\nu$  (true anomaly)) \cite{Chobotov}. The second one, through the calculation of the variation of Equinoctial Orbital Elements (EOE) (p,f,g,h,k,L) \cite{JungHyun}; using for both of them a model based in the Lagrange Planetary Equations \cite{Chobotov}, \cite{JungHyun}. The solution for both methods uses a Runge-Kutta of seventh order.\

Results gives us ideas about the behavior of the methods. This is a joint work with L. Echeverry and J. Soliz  
Key Words: Planetary Lagrange Equations, Orbital perturbations, Orbital elements, Low Earth Orbit (LEO) Satellites.\

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3. Chara Pantazi, Universitat Politècnica de Catalunya, Spain

### **Generalized Darboux Integrals for Schrödinger Planar Vector Fields via Darboux Transformations**

Abstract: In this work we study the Darboux transformations of planar vector fields of Schrödinger type. We use the isogaloisian property of the Darboux transformation in order to explore the "invariance" of the objects of the "Darboux Theory of Integrability" such that the invariant curves, generalized exponential factors, etc. Additionally, we show how the shape invariance property of the potential is important in order to preserve the structure of the transformed vector field. Free particle, three dimensional harmonic oscillator and Coulomb potential, are presented as natural examples coming from supersymmetric quantum mechanics. This is a joint work with and P. Acosta-Humánez.

4. Alberto Reyes-Linero, Universidad del Atlántico, Colombia

**Algebraic and Qualitative Analysis to the Family  $y'=(\alpha x^{2k}+\beta x^{m-k-1})y+\gamma x^{2m-2k-1}$ .**

Abstract: Analysis of dynamical systems has been a topic of great interest to mathematicians and physicists. Each system has their own characteristics, which allows grouping these families and study them. One of these families can be seen in the problem 11 of the section 1.3.3, on Book; Handbook of exact solutions for ordinary differential equations, by Polyanin-Zaitsev, Which is a family with five parameters of Lienard's systems. About this family, inspired by the paper authored by Acosta-Humánez, Lázaro, Morales-Ruiz and Pantazi, a Galoisian study is performed, making a series of transformations (using some tools like the Hamiltonian Algebrization) which allow the Lienard Equation to take the Second Order Equation, Then a Gegenbauer Equation, followed by Hypergeometric equation and finally in a Legendre equation. with help of the Differential Galois theory, allows us to conclude if the system's integrability or not integrability. Finally we will make a study of the qualitative properties of this family, such as conditions so that the system is formed by polynomials functions, study also critical points, conditions for their existence and stability. This is a joint work with P. Acosta-Humánez and J. Rodríguez

5. Henock Venegas, Universidad del Atlántico, Colombia

**Galoisian approach to algebraically solvable and quasi-solvable polynomial potentials**

Abstract: In this work we present some Galoisian aspects of Schrodinger equations with polynomial potentials. In particular, harmonic oscillator (algebraically solvable potential), anharmonic potentials (quartic and sextic algebraically quasi-solvable potentials) are deeply analysed. This is a joint work with P. Acosta-Humánez.

6. Yovani Villanueva, Universidad Sergio Arboleda, Colombia

**Calculus of optimal space trajectories using particle swarm optimization for finding each seed in the multivariate Newton's method**

Following [LH], we've studied a newfangled method for finding the optimum conditions for a (keplerian) fly-by between two celestial bodies, interceptor and target. We look for minimal  $\Delta \vec{v}_0$ , the velocity increment, and the time of fly  $t$  needed for the interceptor to get the target, this is the transfer orbit. The data are the initial positions and velocities of the target and the interceptor [RO]. Then we are going to optimize an energy performance index under the following restrictions: 1. keplearian orbits of the target and the interceptor using universal variables, [RB], [JD] 2. the final positions of them must be the same. As [LH] we use Lagrange multipliers for optimizing but we propose a different development for them, our system has three equations more than the ones used in the Leeghim's article. Also we use the multivariate Newton's method to solve the nine resultant equations.

This technique is a variation of the Lambert problem, but in this case we don't know the motion direction and the time of flight of the transfer at the beginning. Other variation is the optimization of wait time  $t_1$  (the interceptor doesn't leave its initial orbit until the relative positions between the target and that are optimal) together with the variables of above, whose performance is very similar to the first technique. However, the seeds of the Newton's method are very difficult to find, so we used the Particle

Swarm Optimization (\cite{HM} and \cite{CB}) for determining some possible domains where the seeds could stay and after the seeds was exactly found through a seed finder (eight loops with specific partitions and a lot of possibilities such that everyone is evaluated in the Newton's method).\

\noindent In addition, we realized that the PSO can distinguish some topological and analytical characteristics of the problem. Now we are collecting information of the execution of the programs and doing the descriptive and exploratory process (statistical analysis and finding relationships) with the data. \

\noindent These algorithms was applied to low Earth orbits, where the HCW equations are useful \cite{VD}, and was compared with other approximation technics for fly-by whose result was that this method is better. Also we built a table that shows the optimal travel to each two planets of the solar system using this methods and we made some comparisons with the optimization results shown in \cite{KS} (Lambert's problem). This is part of the research support of the project ``Libertad II'', the second satellite of the Sergio Arboleda University. This is a joint work with L. Echeverry and J. Soliz

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